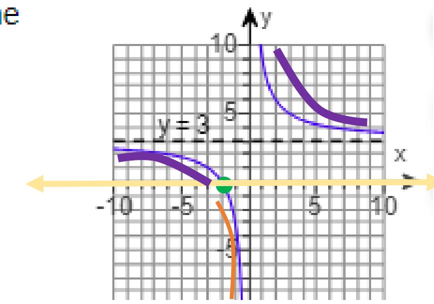
3.3 Rational Inequalities Math 161 THOMPSON

1) The graph of a rational function y = f(x) is given. Use the graph to give the solution set of the following.

(a) f(x) = 0
(b) f(x) < 0
(c) f(x) > 0

Do not use a calculator.

x-axis



(a) What is the solution set of f(x) = 0?

Intervals (-∞,-2) (-2,0) (0, ∞)

+ - +

Crosses the x-axis

A. The solution set is {−2}.
(Type an integer or a decimal. Use a comma to separate answers as needed.)

(b) What is the solution set of f(x) < 0? Below the x-axis

A. The solution set is (−2,0). (Type your answer in interval notation.)

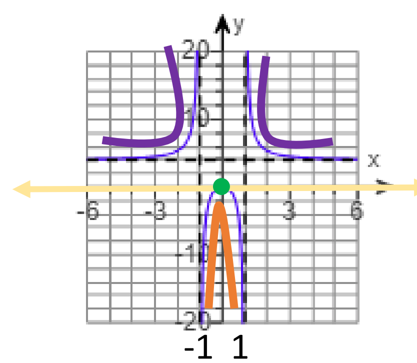
(c) What is the solution set of f(x) > 0? Above the x-axis

A. The solution set is (−∞,−2)∪(0,∞). (Type your answer in interval notation.)

2) The graph of a rational function y = f(x) is given. Use the graph to give the solution set of the following.

(a) f(x) = 0
(b) f(x) < 0
(c) f(x) > 0

Do not use a calculator.



(a) What is the solution set of f(x) = 0?

Crosses the x-axis

Intervals (-∞,-1) (-1,0) (0,1) (1, ∞)

A. {0} (Type an integer or a decimal.)

(b) What is the solution set of f(x) < 0? Below the x-axis

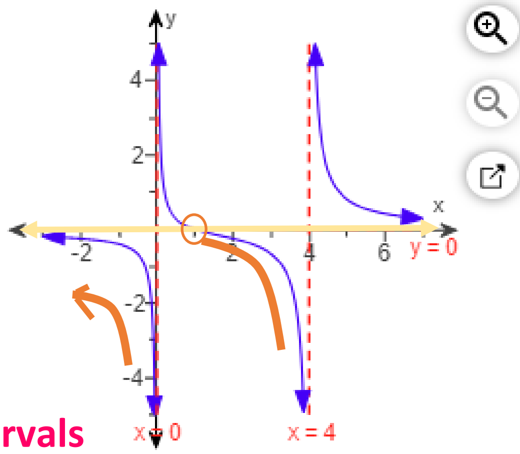
+ - - +

A. (−1,0)∪(0,1)

(c) What is the solution set of f(x) > 0? Above the x-axis

A. (−∞,−1)∪(1,∞)

3) Solve the inequality $R(x) < 0$, where $R(x) = \frac{x-1}{x(x-4)}$, by using the graph of the function.



Intervals

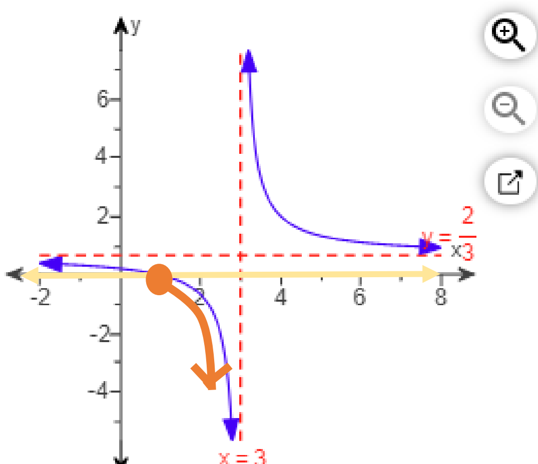
(-∞,0) (0,1)(1,4) (4, ∞)

- + - +

The solution set for $R(x) < 0$ is $(-\infty,0)\cup(1,4)$.
(Type your answer in interval notation.)

Below the x-axis

4) Solve the inequality $R(x) \le 0$, where $R(x) = \frac{2x-2}{3x-9}$, by using the graph of the function.



Intervals

(-∞,0) [1,3) (3, ∞)

+ - +

The solution set for $R(x) \le 0$ is $[1,3)$.
(Type your answer in interval notation.)

Below and ON the x-axis

Includes 1 but not 3

FRACTIONAL INEQUALITIES
MAKE A NUMBER LINE

$\geq$ *means* ●
$>$ *means* ○
all values on bottom are ○

5) Solve the inequality $\frac{x+6}{x-9} < 0$

*we check all the critical points on a number line   x = -6, 9

Intervals   **(-∞,-6)**   **(-6,9)**   **(9, ∞)**

check #s   **-7** *(left of 6)*   **0** *(between -6 & 9)*   **10** *(right of 9)*

$\frac{-7+6}{-7-9} = +$ $\frac{0+6}{0-9} = -$ $\frac{10+6}{10-9} = +$

< Negative interval where both are open   **(-6,9)**

6) Solve the rational inequality and graph the solution set on a real number line. Express the solution set in interval notation.

$\frac{x+3}{x+2} \leq 0$ *we check all the critical points on a number line   x = -3, -2

≤ is closed circle at -3

BOTTOM # is always open circle

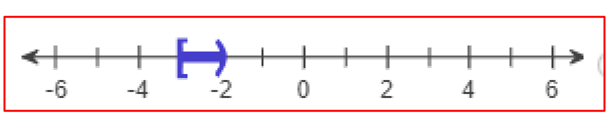
Intervals   **(-∞,-3)**   **(-3,-2)**   **(-2, ∞)**

check #s   **-4** *(left of 6)*   **-2.5** *(between -3 & -2)*   **0** *(right of -2)*

$\frac{-4+3}{-4+2} = +$ $\frac{-2.5+3}{-2.5+2} = -$ $\frac{0+6}{0+2} = +$

≤ Negative interval where -3 is closed, -2 is open

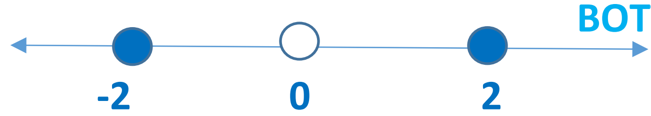
[-3,-2)



7) Solve the inequality $\frac{(x-2)(x+2)}{x} \leq 0$ closed circles from ≤

*we check all the critical points on a number line x = -2, 0, 2

BOTTOM # is always open circle



Intervals **(-∞,-2)** **(-2,0)** **(0,2)** **(2, ∞)**

check #s -3 -1 1 3

$\frac{(-3-2)(-3+2)}{-3} = -$ $\frac{(-1-2)(-1+2)}{-1} = +$ $\frac{(1-2)(1+2)}{1} = -$ $\frac{(3-2)(3+2)}{3} = +$

Interval	$(-\infty,-2)$	$(-2,0)$	$(0,2)$	$(2,\infty)$
Sign	Negative	Positive	Negative	Positive

Intervals are always ( ) in this box

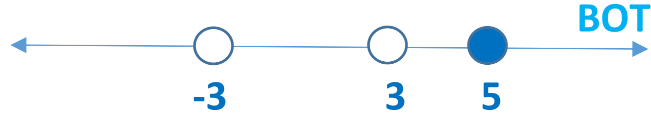
≤ Negative intervals where -2 and 2 are closed, 0 is open

(- ∞,-2] ∪(0,2]

8) Solve the inequality $\frac{(x-5)^2}{x^2-9} \geq 0$

*we check all the critical points on a number line x = -3, 3, 5

BOTTOM #s are always open circle



Intervals **(-∞,-2)** **(-3,3)** **(3,5)** **(5, ∞)**

check #s -4 0 4 6

$\frac{(-4-5)^2}{4^2-9} = +$ $\frac{(0-5)^2}{0^2-9} = -$ $\frac{(4-5)^2}{4^2-9} = +$ $\frac{(6-5)^2}{6^2-9} = +$

≥ Positive intervals where 5 is closed, -3,3 are open

(-∞,-3)∪(3, ∞)

9) Solve the inequality $\frac{x^2(9+x)(x-9)}{(x+2)(x-5)} \geq 0$ closed circles from ≥

*we check all the critical points on a number line x = -9, -2, 0, 5, 9

BOTTOM #s are always open circle

Intervals (-∞,-9) (-9,-2) (-2,0) (0,5) (5,9) (9, ∞)

check #s -10 -3 -1 2 6 10

+ - + + - +

≥ Positive interval where -9,0,9 are closed, -2,5 are open

(-∞,-9]∪(-2,5) ∪[9,∞)

WORK FOR CHECKING #S

-10 $\frac{10^2(9-10)(-10-9)}{(-10+2)(10-5)}$ + -3 $\frac{3^2(9-3)(-3-9)}{(-10+2)(10-5)}$ − -1 $\frac{10^2(9-1)(-1-9)}{(-10+2)(10-5)}$ +

2 $\frac{2^2(9+2)(2-9)}{(2+2)(2-5)}$ + 6 $\frac{6^2(9+6)(6-9)}{(6+2)(6-5)}$ − 10 $\frac{10^2(9+10)(10-9)}{(10+2)(10-5)}$ +

10) Solve the inequality $\sqrt{\frac{x-10}{x+6}}$

square root means x ≥0 but bottom cannot = 0 so bottom x > 0

*we check all the critical points on a number line $x = -6, 10$

Open at -6 because bottom of fraction

Intervals (-∞,-6) (-6,10) (10, ∞)

check #s -7 0 11

$\sqrt{\frac{-7-10}{-7+6}}$ + $\sqrt{\frac{0-10}{0+6}}$ **undefined** $\sqrt{\frac{11-10}{11+6}}$ +

≥ Positive intervals where 10 is closed and -6 is open

(-∞,-6)∪[10, ∞)

Another 10) Solve the inequality $\sqrt{\frac{x-8}{x+4}}$

square root is $x \geq 0 \rightarrow$ bottom cannot = 0; **Open at -4 because bottom of fraction**

*we check all the critical points on a number line $x = -4, 8$

check #s -5 0 9

\+ - +

≥ **Positive intervals where 8 is closed and -4 is open**

$(-\infty,-4)\cup[8,\infty)$

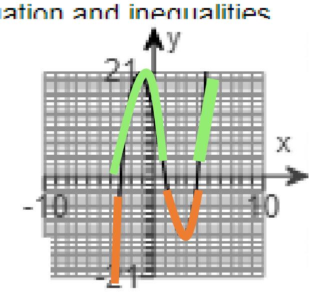
EXTRA EXAMPLES

A) Graph the following polynomial function by hand. Then solve the equation and inequalities

$P(x) = x^3 - 2x^2 - 11x + 12$

$= (x-4)(x-1)(x+3)$ crosses at -3,1,4

(a) $P(x) = 0$ **(b)** $P(x) < 0$ **(c)** $P(x) > 0$



on x axis

(a) The solution set for $P(x) = 0$ is $\{-3,1,4\}$.

(Use a comma to separate answers as needed.)

(b) The solution set for $P(x) < 0$ is $(-\infty, -3) \cup (1, 4)$. Graph below the x-axis

(Type your answer in interval notation.)

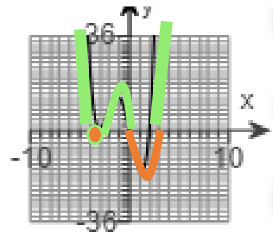
(c) The solution set for $P(x) > 0$ is $(-3,1) \cup (4,\infty)$. Graph above the x-axis

(Type your answer in interval notation.)

B) Graph the following polynomial function by hand. Then solve the equation and inequalities.

$P(x) = x^4 + 4x^3 - 3x^2 - 18x$

$= x(x-2)(x+3)^2$ crosses at 0,2 touches at -3



(a) $P(x) = 0$ **(b)** $P(x) \geq 0$ **(c)** $P(x) \leq 0$

on x axis

(a) The solution set for $P(x) = 0$ is $\{-3,0,2\}$.
(Use a comma to separate answers as needed.)

(b) The solution set for $P(x) \geq 0$ is $(-\infty,0] \cup [2,\infty)$. Graph on or above the x-axis
(Type your answer in interval notation.)

(c) The solution set for $P(x) \leq 0$ is $\{-3\} \cup [0,2]$. Graph on or below the x-axis
(Type your answer in interval notation.)

C) Solve the inequality. Express your answer using set notation or interval notation. Graph the solution set.

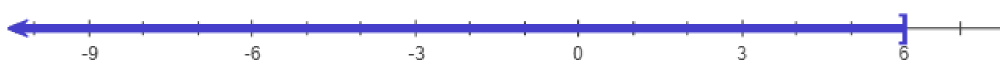
$17 - 3x \geq -1$

Choose the correct answer below that is the solution set to the inequality.

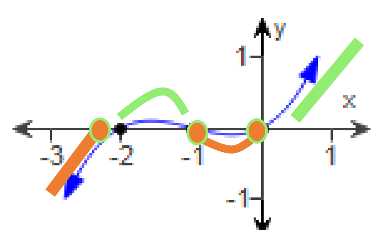
$-3x \geq -18$

$x \leq 6$ *change direction dividing by -3*

- A. $\{x | x \leq 6\}$ or $(-\infty, 6]$
- B. $\{x | x \leq -6\}$ or $(-6, \infty)$
- C. $\{x | x \leq -18\}$ or $(-18, \infty)$
- D. $\{x | x \geq 6\}$ or $[-\infty, 6)$



D) Use the graph of the function f to solve the inequality.



(a) $f(x) > 0$ $(-2,-1) \cup (0,\infty)$

(b) $f(x) \leq 0$ $(-\infty,-2] \cup [-1,0]$

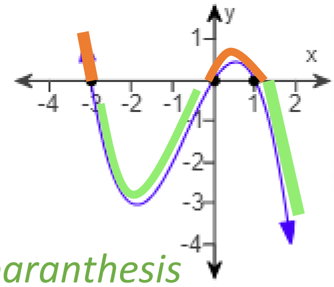
*a) *since it is >, it does not include the point on the x-axis and parenthesis*

*b) *since it is ≤, it does include the point on the x-axis and brackets*

E) Use the graph of the function f to solve the inequality.

(a) f(x) < 0 $(-3,0)\cup(1,\infty)$

(b) f(x) ≥ 0 $(-\infty,-3]\cup[1,1]$



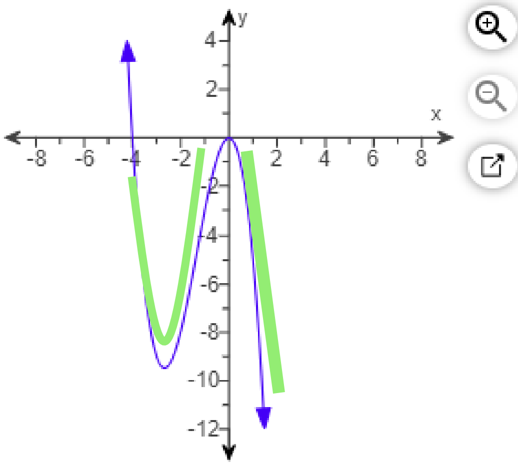
a) *since it is <,*

below (not on) the x-axis paranthesis

b) **since it is ≥,*

on and above the x-axis and brackets

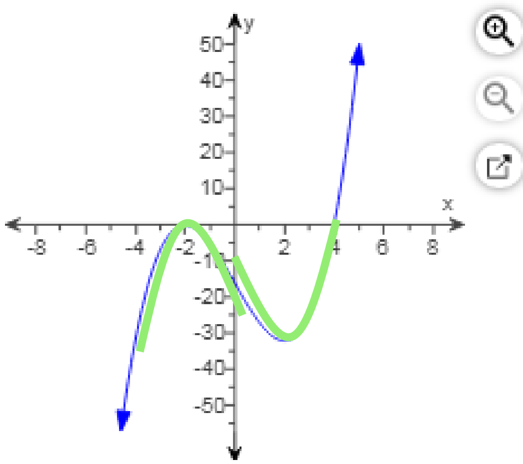
F) Solve the inequality f(x) < 0, where $f(x) = -x^2(x+4)$, by using the graph of the function.



(-4,0)U(0,∞)

since it is <, below the x-axis

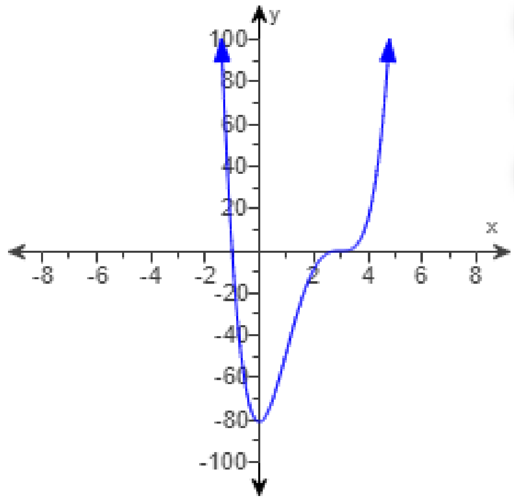
G) Solve the inequality f(x) ≤ 0, where $f(x) = (x-4)(x+2)^2$, by using the graph of the function.



$(-\infty,4]$

since it is ≤, below and on the x-axis

H) Solve the inequality $f(x) \geq 0$, where $f(x) = 3(x+1)(x-3)^3$, by using the graph of the function.



$(-\infty,-1] \cup [3, \infty)$

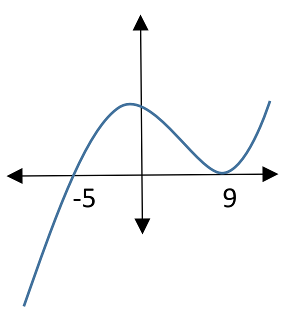
I) Solve the inequality algebraically.

$(x-9)^2(x+5) < 0$ Intercepts are -5 and 9

List the intervals and sign in each interval. Complete the following table.
(Type your answers in interval notation. Use ascending order.)

Interval	$(-\infty,-5)$	$(-5,9)$	$(9,\infty)$
Sign	Negative	Positive	Positive

*easiest to make a graph of x^3 that touches at 9 crosses at -5



Use this graph to fill in the positive or negative in the chart

***below** the x-axis since it is < 0*

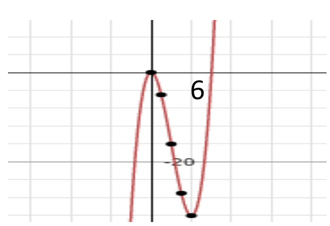
$(-\infty,-5)$

J) Solve the inequality $x^3-6x^2 > 0$ *factor first*

$x^2(x-6) > 0$ *x-intercepts are 0 and 6*

*easiest to make a graph of x^3 that touches at 0 and crosses at 6

***above** the x-axis since it is > 0*

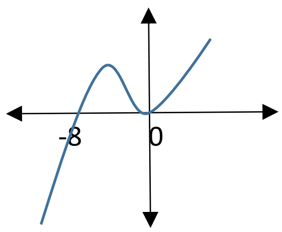


$(6,\infty)$

K) Solve the inequality $3x^3 > -24x^2$ $3x^3+24x^2 > 0$ *factor first*

$3x^2(x+8) > 0$ *x-intercepts are 0 and -8*

*easiest to make a graph of x^3 that touches at 0 and crosses at -8



Interval	$(-\infty,-8)$	$(-8,0)$	$(0,\infty)$
Sign	Negative	Positive	Positive

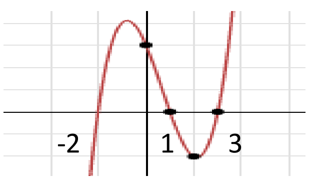
***above** the x-axis since it is > 0*

open circle at 0

$(-8,0) \cup (0,\infty)$

L) Solve the inequality $(x-3)(x-1)(x+2) \geq 0$ *x-intercepts are -2, 1 and 3*

*easiest to make a graph of x^3 that crosses at -2, 1, 3



***above and on** the x-axis since it is ≥ 0*

$[-2, 1] \cup [3, \infty)$

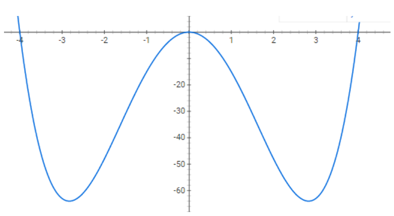
M) Solve the inequality $x^4 > 16x^2$ *solve first then factor*

$x^4-16x^2 > 0$

$x^2(x^2-16)>0$

$x^2(x+4)(x-4)>0$ *x-intercepts are -4, 0 and 4*

*easiest to look at the graph of x^4 and touches at 0 crosses at -4 and 4



above the x-axis since it is > 0

$(-\infty,-4) \cup (4, \infty)$

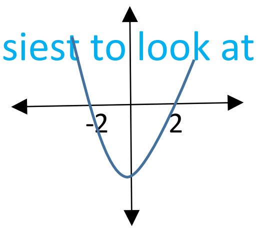
N) Solve the inequality $x^4>16$ *solve first then factor*

$x^4-16>0$

$(x^2-4)(x^2+4)>0$

$(x-2)(x+2)(x^2+4)>0$ *x-intercepts are -2, 2*

*easiest to look at the graph of x^4 and crosses at -2 and 2



above *the x-axis since it is > 0*

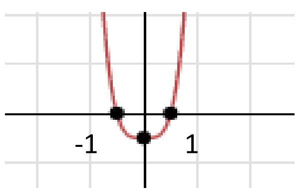
$(-\infty,-2)\cup(2,\infty)$

O) Solve the inequality $\sqrt{x^4-1}$ square root means $x \geq 0$

$(x^2-1)(x^2+1)$

$(x-1)(x+1)(x^2+1)$

*easiest to look at the graph of x^4 and crosses at -1,1



above and on the x-axis since it is ≥ 0

$(-\infty,-1]\cup[1,\infty)$

P) Solve the quadratic inequality. Give the solution set in interval notation.

$(x+8)^2 \leq 0$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The solution set is the single point $\{-8\}$.
(Type an integer or a simplified fraction.)

Q) Solve the inequality (x-3)(x-1)(x+2)> 0

*easiest to look at the graph of x^3 and crosses at -2, 1, 3

above the x-axis since it is > 0

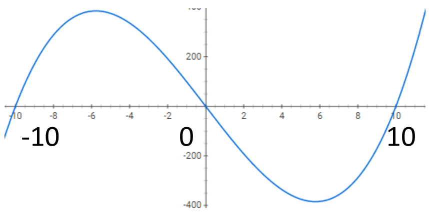
(-2, 1)∪(3, ∞)

R) Solve the inequality x^3-100x ≤ 0

$x(x^2-100) \leq 0$

$x(x-10)(x+10) \leq 0$

*easiest to look at the graph of x^3 and crosses at -10, 0, 10



below and include the x-axis since it is < 0

(-∞,-10]∪[0,10]

S) Solve the inequality $9x - 9 \geq -4x^2$

$4x^2 + 9x - 9 \geq 0$ *use slide and divide*

$x^2 + 9x - 36$

$(x + 12)(x - 3)$ *divide by 4* $x = -3, \frac{3}{4}$

*we check all the critical points on a number line $x = 3, -\frac{3}{4}$

Open at zero because bottom of fraction

check #s	-4	0	1
	+	-	+

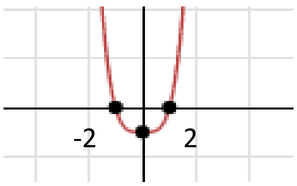
all positive intervals

(-∞,-3]∪[$\frac{3}{4}$, ∞)

T) Solve the inequality $\sqrt{x^4-16}$ *square root means x ≥0*

$(x^2\text{-}4)(x^2\text{+}4)$

$(x\text{-}2)(x\text{+}2)(x^2\text{+}1)$



*easiest to look at the graph of x^4 and crosses at -2,2

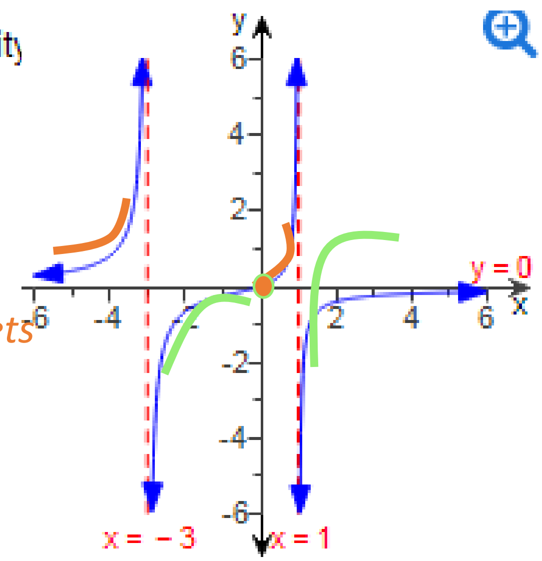
We are looking for above and include the x-axis since it is ≥0

(-∞,-2]∪[, ∞)

U) Use the graph of the function f to solve the inequality

(a) f(x) < 0 (-3,0)∪(1, ∞)

(b) f(x) ≥ 0 (-∞,-3)∪[0,1)



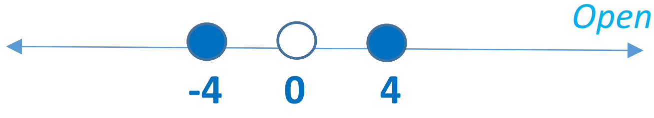
*a) *since it is <, below the x-axis and parenthesis*

*b) *since it is ≥, on and above the x-axis and brackets*

V) Solve the inequality $\frac{(x-4)(x+4)}{x} \geq 0$

*we check all the critical points on a number line x = -4, 0, 4

Open at zero because bottom of fraction



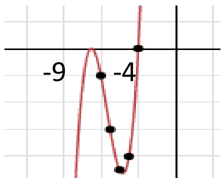
check #s -5 -1 1 5

\- + - + *all positive intervals*

[-4,0)∪[4, ∞)

W) Solve the inequality $(x+9)^2(x+4) < 0$ *x-intercepts are -9 and -4*

*easiest to make a graph of x^3 that touches at -9 crosses at -4



below the x-axis since it is < 0

$(-\infty,-9)\cup(-9,-4)$